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Cognitive Mechanisms of Insight: The Role of Heuristics and Representational Change in Solving the Eight-Coin Problem

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The 8-coin insight problem requires the problem solver to move 2 coins so that each coin touches exactly 3 others. Ormerod, MacGregor, and Chronicle (2002) explained differences in task performance across different versions of the 8-coin problem using the availability of particular moves in a 2-dimensional search space. We explored 2 further explanations by developing 6 new versions of the 8-coin problem in order to investigate the influence of grouping and self-imposed constraints on solutions. The results identified 2 sources of problem difficulty: first, the necessity to overcome the constraint that a solution can be found in 2-dimensional space and, second, the necessity to decompose perceptual groupings. A detailed move analysis suggested that the selection of moves was driven by the established representation rather than the application of the appropriate heuristics. Both results support the assumptions of representational change theory (Ohlsson, 1992).

Keywords: insight, heuristics, representational change, problem solving

Some problems initially appear trivial yet prove to be very difficult and time-consuming. They often have no obvious solution, and solution strategies used in the past cannot successfully be applied to them. Sometimes, a sudden, unintended, and unexpected solution appears, often accompanied by an aha experience. Such problems are termed *insight problems* (Dominowski & Dallob, 1995; Öllinger & Knoblich, 2009). Two different approaches explain the dynamics of insight problem solving as an extension of problem space theory (Newell & Simon, 1972): Ohlsson's representational change theory (RCT; Ohlsson, 1992) and MacGregor and colleagues criterion for satisfactory progress theory (CSPT; MacGregor, Ormerod, & Chronicle, 2001).

According to RCT, prior knowledge determines which problem elements are parts of a problem representation. Perceptual processes will group some or all of the problem elements into meaningful chunks, and problem solvers will form a particular representation of the expected goal. The goal representation constrains the initial exploration of the problem space.

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Accordingly, in Ohlsson and colleagues' framework (Knoblich, Ohlsson, Haider, & Rhenius, 1999; Ohlsson, 1992), *chunk decomposition* and *constraint relaxation* are two key mechanisms for changing an overconstrained representation and attaining insight. Both have been addressed in earlier empirical studies. Chunk decomposition (Knoblich et al., 1999) is a process at the interface of perception and conception that can assign new meanings to perceptual elements of a problem representation. Once a chunk is decomposed, new chunks can be formed, and new solution paths can be considered (Knoblich, Ohlsson, & Raney, 2001; Öllinger, Jones, & Knoblich, 2006, 2008). Constraint relaxation changes the representation of the goal of the problem-solving process. This is needed to overcome the self-imposed constraints that prevent a problem solver from finding the correct solution.

Whereas RCT focuses on prior knowledge, CSPT extends more directly the assumptions of problem space theory (Newell & Simon, 1972). The theory starts with the assumption that problem spaces are often too large to be fully explored, and appropriate heuristics that guide and restrict the search space are often missed (Kaplan & Simon, 1990). CSPT proposes that the balanced interplay of different heuristics is a crucial component for the solution of many insight problems.

In particular, MacGregor et al. (2001) proposed that solution of the nine-dot problem and similar insight problems is attained by combining a *maximization heuristic* (hill climbing) and a *progress monitoring* heuristic. The maximization heuristic drives the selection of moves by reducing, as much as possible, the distance between the current state and the goal state. At the same time, the progress monitoring heuristic is used to compute the ratio between the progress made by a selected move and the potential of the remaining moves to attain the goal. When problem solvers use the two heuristics to realize that the problem cannot be solved, they start looking for new and more promising states. Accordingly, an insight in the solution of a problem can be driven by the realization that still unexplored parts of a problem play a crucial role for the solution. In general, the CSPT focuses on the search process, whereas the RCT focuses on the initial representation that was activated by prior knowledge.

Ormerod, MacGregor, and Chronicle (2002) provided further evidence in support of the CSPT using the eight-coin problem. The goal in the eight-coin problem is to find a configuration in which each of eight coins touches exactly three other coins by moving two coins (see Figure 1). The correct solution requires mounting two single coins on top of two separate groups of three coins (see Figure 1b), –that is, extending the problem space from "2-D" to "3-D" and decomposing the given coins into two groups.

According to the CSPT, the difficulty of a problem depends on how many moves meet a particular maximization criterion. The maximization criterion in the eight-coin problem can be defined as maximizing the number of coins that touch exactly three other coins. Perceived progress toward the solution can be defined as the number of moves that satisfy the criterion that a coin touches three other coins (Ormerod et al., 2002, p. 793). When limited moves satisfy such a criterion, the problem solver should quickly realize the need to look for new avenues to explore; when many moves satisfy the criterion, exploration of alternative moves will take place more slowly. The problem depicted in Figure 1a does not offer any moves within a 2-D representation that will make one coin touch exactly three others, thus a search for promising new states should begin quickly. The problem in Figure 1c allows such moves. CSPT therefore predicts lower solution rates for this problem. This is exactly what Ormerod et al. (2002) found in a pilot study, leading to further studies designed to test competing predictions of the CSPT and RCT.

The RCT assumes that either perceptual grouping factors (chunk decomposition) and/or overcoming the 2-D constraint (constraint relaxation) trigger a representational change that permits the solution of the eight-coin problem. Ormerod et al. (2002) ruled out this claim: In their first experiment, a 2 × 2 design introduced problems with either tight or loose perceptual groupings (a loose perceptual grouping being a grouping that is easily broken down into its constituent parts, whereas a tight grouping is not), and either 2-D criterion moves were available or not (these problems were variants of those seen in Figure 1). Crucially, 79% of participants in the "no criterion moves available" condition (a problem analogous to that of Figure 1a, representing early criterion failure) solved the problem, whereas only 50% of the participants in the "criterion moves available" condition solved the problem. Tight or loose perceptual groupings had no influence on solution rates.

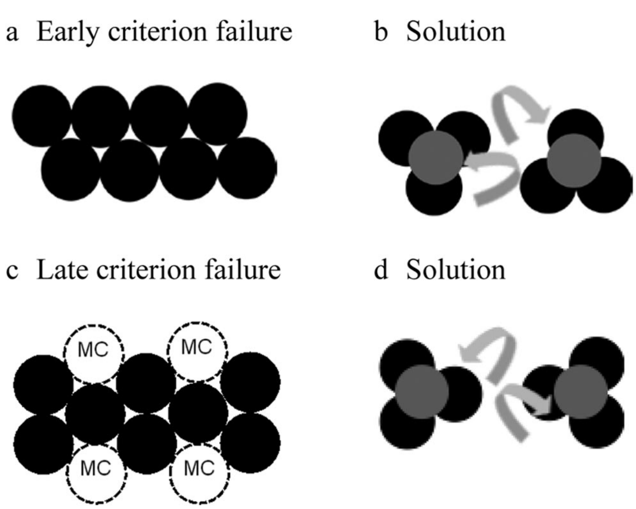


Figure 1. a: Problem without criterion moves in 2-D. b: Solution of Problem 1a. c: Problem having moves that meet the criterion, where the white circles indicate locations where a first move can meet the criterion (MC). d: Solution of Problem 1c.

In the second experiment, Ormerod et al. (2002) addressed whether providing 3-D cues increased solution rates. Problems either included a visual 3-D cue (one coin stacked vertically onto another coin, relaxing the 2-D constraint) or not. The results showed no influence of the visual hint; solution rates depended on whether 2-D criterion moves were available or not.

The aim of the present study was to further clarify the influence of grouping and 3-D cues on the solution of the eight-coin problem. Six new eight-coin problems were developed that allowed the grouping and 3-D perceptual characteristics to be varied. Table 1 shows these problems (Problems C–H), together with the original eight-coin problems as introduced by Ormerod and colleagues (2002; Problems A and B).

Problem B is the only one of the eight problems that provides available moves that meet the "three coins matching" criterion in 2-D. We operationalized the tightness of perceptual grouping as the number of contacts between the coins of a certain problem. That is, we vary the total number of coins that are in contact with one another across the eight problems. This follows the same logic as Ormerod and colleagues' (2002) manipulation of chunk tightness with the exception that their measure also included symmetry. Additionally, we introduced three problems that relaxed the perceived 3-D constraint to investigate the influence of a perceptual 3-D cue on solution rates.

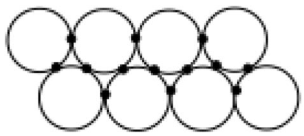
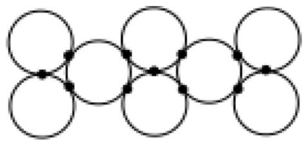
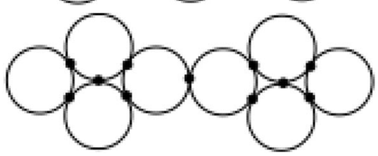
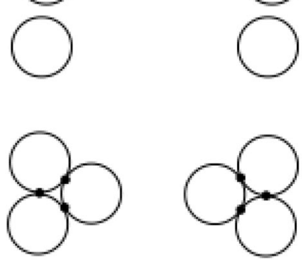
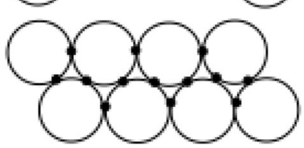
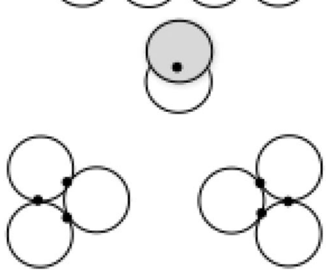
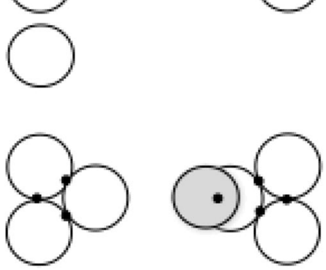
Hypotheses

Problem Difficulty

RCT predicts that two factors should contribute to an overly constrained problem representation in the eight-coin problem: first, perceptual grouping; second, the presence of a 3-D cue. The lower the total number of coins that touch one another (i.e., problems having "loose" chunks), the higher the subsequent solution rates should be for those problems compared with ones where a high number of coins touch one another (i.e., problems having "tight" chunks). The presence of 3-D cues should relax the constraint of 2-D representations, and hence an increase in solution rates should be seen for the 3-D cue problems.

The CSPT predicts that Problem B will be the most difficult due to the fact that it is the only problem that allows 2-D criterion moves. In line with Ormerod and colleagues (2002), the CSPT predicts no effect of 3-D cues and no differences between the perceptual grouping variations.

Table 1
Variation of Grouping and 3-D Perceptual Characteristics With Six New Eight-Coin Problems

	Problem	3-D hint	TNOC	No. groups	2-D CM
A		−	13	1	0
B		−	11	1	4
C		−	11	1	0
D		−	10	2	0
E		−	6	4	0
F		+	9	1	0
G		+	7	3	0
H		+	7	3	0

Note. The first and second columns show the problem label and a visual depiction of the initial configuration. The dots indicate where one coin touches another. The third column depicts whether the problem hints at a 3-D solution (+) or not (−). The fourth column shows the total number of contacts (TNOC) between coins in the start configuration. The fifth column shows the number of separate groups of coins. The sixth column lists the number of possible moves that meet the criterion in a 2-D representation. 3-D = three-dimensional; No. = Number of; 2-D = two-dimensional; CM = criterion move.

Move Selection

For any particular problem, the RCT predicts that coins that are part of loose chunks (i.e., coins that touch few other coins) will be manipulated more often than coins that are part of tight chunks (i.e., coins that touch many other coins) (Knoblich et al., 1999; Öllinger et al., 2006). The RCT further predicts that providing 3-D cues will increase the number of 3-D moves (due to relaxing the 2-D constraint). The CSPT predicts that moves will be selected that maximize the number of coins that touch exactly three others.

Method

Participants

Two hundred twenty-four paid participants (68 men, $M = 25$ years, $SD = 3.67$) were randomly assigned to one of eight experimental groups (28 participants per group).

Material

The eight different versions of the eight-coin problem shown in Table 1 were used. The "coins" were eight circular wooden discs, 18 mm in diameter and 3 mm high.

Procedure

Participants were tested individually in a quiet room. They received the following written instruction (in German): "Your task is to move two coins in such a way that each coin touches exactly three other coins." A printed version of the initial state of the problem was provided in addition to the wooden coins. After each unsuccessful solution attempt, participants were asked to recreate the start configuration, and to start with the next attempt. Participants were allowed as many attempts as they wished.

The participants' workspace was filmed using a camera. The camera was connected to a monitor behind a dividing wall that shielded the experimenter from the participant. The experimenter encoded the starting position and the target position of each move as they were performed. This was done by clicking a mouse on the respective locations of the problem configuration displayed on a computer monitor. The computer recorded the start and end point of each move.

There were no additional written or verbal hints during the experiment. Two experimenters carried out the study. Both were blind to the experimental design.

Results

The structure of the results section is as follows. We first address the prediction of the CSPT that problems where criterion moves were available will be solved less often than problems where criterion moves were unavailable. We then examine solution rates and report an analysis of the coins that were moved in each problem to test the predictions of the RCT for 3-D cues (relaxing the 2-D constraint) and perceptual grouping. As the hypotheses imply, there are at least two ways in which perceptual grouping can be defined in eight-coin problems. One definition is at the level of each individual coin in a problem, which we call a *local* definition. For each problem, there are some coins that can be considered to be loosely chunked and other coins that can be considered to be tightly chunked. This is relative to other coins in the same problem. For example, if Coin Co1 touches four others yet Coin Co2, of the same problem, touches two others, then Coin Co2 is more loosely chunked than Coin Co1. Our local definition of perceptual grouping is therefore the number of contacts (NOC) for each coin within a particular problem. A second way of defining perceptual grouping is at the level of each problem, which we call a *global* definition. Each problem may differ in the total number of contacts between coins, with a low number of contacts reflecting a problem that is loosely chunked and a high number of contacts reflecting a problem that is tightly chunked. Our global definition of perceptual grouping is therefore the total number of contacts (TNOC) between coins in a particular problem. Note that TNOC is a measure that reflects both local properties—how strongly single coins are linked to their neighbors (NOC)—and global properties, such as how many separate groups of coins a problem consists of. In Table 1, for example, Problem A (TNOC = 13) has only one single group, whereas Problem E (TNOC = 6) consists of four separate groups of coins. There is a high negative significant correlation between the TNOC and the number of groups (see Table 1), $r(224) = -.88$, $p < .01$, demonstrating that the more separate groups a problem has, the lower the TNOC.

Criterion Moves

We compared the solution rate of Problem B (criterion moves available in a 2-D representation) with the other 2-D problems (no criterion moves in 2-D). As Table 2 demonstrates, none of the pairwise chi-square test comparisons revealed a significant difference in solution rates with Problem B. That is, we could not replicate the findings of Ormerod and colleagues (2002). The one problem that approaches significance is Problem E. Note that although there are no criterion moves in 2-D for Problem E, it also has fewer contacts between coins (more loose chunks; see Table 1), predicting a greater solution rate under RCT.

3-D Cues

Figure 2 shows the mean solution rate in percent for all versions of the eight-coin problem. The 2-D problems (A–E; see also Table 2) showed lower solution rates than 3-D problems (mean/standard deviation for the 3-D Problems F–H. F: [0.71/0.46]; G: [0.93/0.26]; H: [1.0/0.0]). A chi-square test between the 2-D and 3-D problems revealed a highly significant difference, $\chi^2(1, 224) = 35.34$, $p < .01$, $\lambda = .06$, demonstrating that relaxation of the 2-D constraint causes a significant increase in solution rates.

Table 2
Comparison of Problem B to All Other 2-D Problems

Problem	A	C	D	E
B	.00 (1.00)	.65 (.42)	.65 (.42)	3.50 (.06)

Note. The cells show chi-square and p values (in parentheses). The respective solution rates were as follows: Problem B: 11 out of 28 solved the problem (mean solution rate = 0.39, $SD = 0.50$). A: 11/28 ($M = 0.39$, $SD = 0.50$), C: 14/28 ($M = 0.50$, $SD = 0.51$); D: 14/28 ($M = 0.50$, $SD = 0.51$); E: 18/28 ($M = 0.64$, $SD = 0.49$).

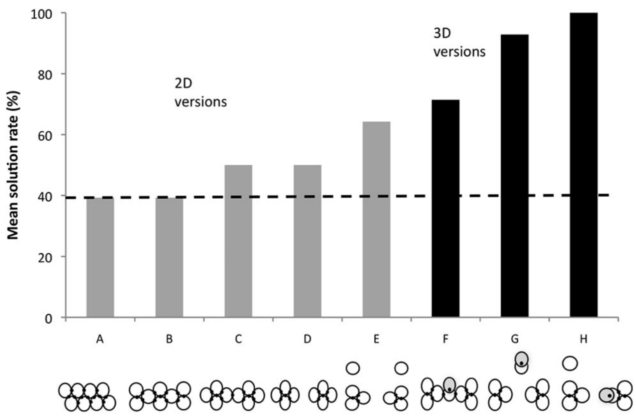


Figure 2. Mean solution rates for all problems. Two-D versions bars are gray; 3-D versions bars are black. The dashed line indicates the performance of Problem B as baseline.

Perceptual Grouping

Figure 3 plots the solution rates as a function of TNOC. The graph shows that problems having a low TNOC have higher solution rates.

In order to test statistically whether TNOC affected solution rates in addition to the effect of 3-D cues, we conducted a two-step binary logistic regression (BLR; Hosmer & Lemeshow, 2000). In particular, we used BLR to determine whether the probability of solving eight-coin problems depended on the number of contacts and, at the same time, on whether a 3-D cue was present or not by estimating weights reflecting to what extent each of the two predictors contributed to the probability that a participant belongs to the "solver" or "nonsolver" categories. The odds ratio derived from the model represents the probability ratio of the criterion (P[Y = 1]/P[Y = 0]; 1 = solved; 0 = not solved) when a predictor is changed.

We first analyzed the main effects of the predictors on solution rates. We entered two predictor variables into the model: (a) the discrete predictor TNOC and (b) the dichotomous predictor 3-D cue present/not present. The reference category for the predictor 3-D cue was the category 3-D cue "not present." A significant effect was obtained in this analysis, $\chi^2(2, 224) = 46.07$, $p < .01$, with the model being able to correctly classify 68.8% of the participants. The analysis revealed a negative influence of the predictor TNOC ($\beta_{TNOC} = -.20$; $Z = 6.69$ [Wald's test], $p < .05$). That is, a higher TNOC significantly reduced the solution rate (see Figure 3). The odds ratio was .82.

The BLR can also further clarify the benefit of the 3-D cue. There was a highly significant effect of 3-D cue ($\beta_{3DC} = 1.57$; $Z = 13.84$, $p < .01$), with an odds ratio of 4.82. The odds ratio of 4.82 means that the model predicts a change in the probability ratio of almost five times between the odds of solving the problem when a 3-D hint is present and the odds of solving the problem when no such hint is present, under the assumption that no other predictor has an influence. In a second step, we analyzed the interaction between both predictors. The interaction model revealed a significant interaction ($\beta = -1.04$; $Z = 3.48$, $p < .05$). The percentage of correctly classified participants did not change after adding the interaction term (68.8%). Because the model did not improve, we do not interpret the interaction further.

Move Analysis

We conducted the move analysis in two steps. First, following Ormerod et al. (2002), we examined the first move of each solution attempt by determining the preferred *start* and *end* positions of a move. Any initial move that formed part of the final solution was excluded. The goal was to identify whether the number of criterion moves available affected which coin was selected for the initial move and to identify whether the presence of loose chunks affected which coin was selected for the initial move. Second, we examined all moves that involved stacking one coin on top of another in order to identify whether the presence of a 3-D cue influenced the selection of moves that involved 3-D space rather than 2-D space. Figure 4 shows the frequency by which each coin was selected to be the first move, together with any end positions that were selected as an ending point of a move significantly more often than chance.

Criterion Moves

There is one aspect of Figure 4 that clearly illustrates how participants search for moves that satisfy the criterion: All of the end positions of the first move in Problem B result in the moved coin touching three and only three others. In this respect, the CSPT is supported—although as we have seen earlier, the prediction that all other problems would be of equal difficulty (because none offer 2-D criterion moves) was not borne out. This implies that although the number of criterion moves available affect problem behavior, participants were able to quickly recover because the solution rates to Problem B were no different from those for Problems A, C, D, and E.

Perceptual Grouping

With regard to the RCT, Figure 4 indicates that participants preferred to move coins that were part of loose chunks—and

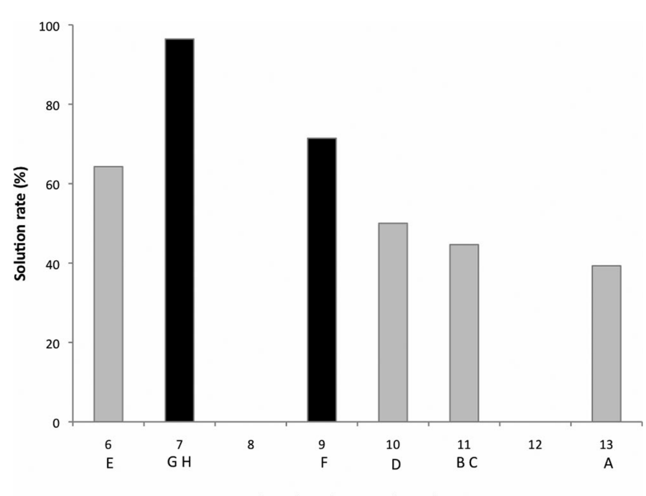


Figure 3. Solution rate as a function of the total number of contacts separated for 2-D (gray bars) and 3-D versions (black bars). The letters on the *x*-axis indicate the problem versions.

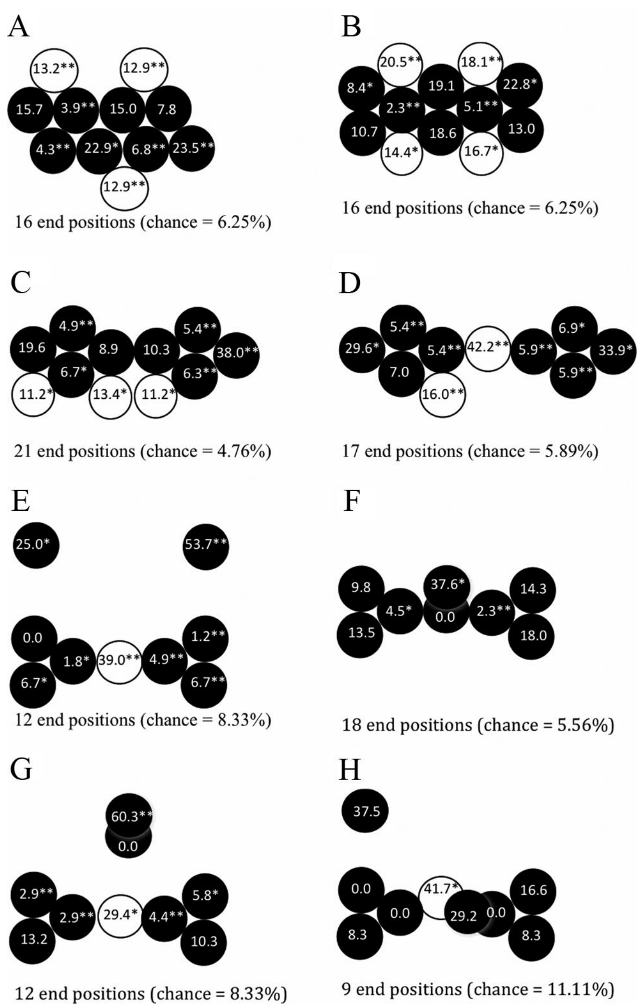


Figure 4. Percentage of all start positions (filled circles) and those end positions that were chosen significantly above chance (unfilled circles). For the start position, 12.5% was set as the test value because this is the chance probability of selecting one of the eight coins; one-sample t tests were conducted. Significant values below 12.5% indicate that coins were selected significantly less often than expected. The number of end positions was determined empirically for each problem by counting the number of each position (NP) that at least one participant had chosen. The baseline was computed by 1/NP. * $p < .05$. ** $p < .01$.

particularly those coins that were isolated (Problems E, G, and H). In order to verify this statistically, we first grouped coins together according to the factor NOC. For each problem, we determined the NOC for each individual coin. As Table 3 shows, the NOC for coins varied from 0 to 4. Within each problem, coins with the lowest NOC were considered to be part of loose chunks and coins with the highest NOC to be part of tight chunks (i.e., we ignored intermediate chunks, because not all problems had intermediate chunks). For example, coins in Problem A touched two, three, or four other coins. Coins touching two others were therefore considered to be part of a loose chunk, and coins touching four others were part of a tight chunk. For Problem E, however, coins either touched zero other coins or touched two other coins. Coins touching zero others were therefore considered loose chunks for this problem, and coins touching two others were considered tight chunks. Second, we computed the dependent variable as the mean frequency of selection for coins at each NOC. For example, if a problem had three coins having NOC = 2, and the number of times those coins were selected as the first move was six, then a coin with NOC = 2 was selected, on average, two times (see Table 3).

The analyses included only participants who selected both loose and tight chunks. A repeated measures analysis of variance (ANOVA) with the factor Chunk (loose, tight) revealed a highly significant effect, $F(1, 177) = 93.32$, $\eta_p^2 = .34$, $p < .01$ (mean number of times a coin involved in a loose chunk was selected = 1.84; $SD = 1.83$; for tight chunks, $M = 0.54$, $SD = 0.69$). Table 3 shows the differences between loose and tight chunks for each problem.

The Mean column of Table 3 indicates that, in general, the NOC influences the frequency of coin selection. A regression analysis with the predictor NOC (0, 1, 2, 3, 4) and the criterion weighted mean number of selected coins (see Table 3) showed a negative but significant influence of NOC ($\beta = -.38$), $t(451) = 6.20$, $p < .01$, and explained a significant amount of the variance ($R^2 = .08$), $F(1, 451) = 38.39$, $p < .01$, showing that, in general, participants preferred moving coins with fewer contacts.

Finally, it is possible that the effect of NOC is not due to the NOC between coins but is driven by more general perceptual grouping aspects. In particular, Problems E, G, and H all include isolated groups of coins that might drive move selection. To ensure this was not the case, we confirmed that a strong effect of NOC exists even when Problems E, G, and H are excluded from the analysis, $F(1, 124) = 59.60$, $p < .01$, $\eta_p^2 = .33$. That is, even when problems with isolated coins are removed from the analysis, there is still a higher likelihood to manipulate loose chunks than tight ones.

3-D Moves

We analyzed whether providing a 3-D cue increases the probability of moving coins to "3-D" positions. In total, 159 of 224 participants applied 3-D moves. Out of the 159 participants, 142 (89%) solved the problem. To assess the influence of 3-D cues on the selection of 3-D moves, we created two groups by pooling and then averaging the 3-D move data of Problems F, G, and H in the 3-D cue present category and the 3-D move data of the remaining five problems (A, B, C, D, and E) in the 3-D cue not present category (see Figure 5).[1] We conducted a one-way ANOVA with the factor 3-D Cues (present, not present). The analysis revealed a highly significant main effect, $F(1, 222) = 50.87$, $p < .01$, $\eta_p^2 = .19$, showing that presenting 3-D cues significantly increased the number of selected 3-D moves.

Eight of the 10 participants who failed to solve a 3-D problem belonged to Problem F, which has a tighter perceptual grouping of the eight coins compared with Problems G and H (TNOC = 9 for Problem F; TNOC = 7 for Problems G and H). A chi-square test

[1] For the 3-D cue problems, one may expect the lowest number of 3-D moves to appear in Problem F (because this problem has the least number of solvers of Problems F, G, and H). However, as Figure 4 shows, a large number of moves in Problem G involved closing the existing gap in a 2-D representation, thus driving down the number of 3-D moves that people make for this problem.

Table 3
Weighted Mean Number of Selected Coins That Have Zero to Four Contacts

NOC	A	B	C	D	E	F	G	H	*M*
0	—	—	—	—	*2.69***	—	—	*.82**	1.75
1	—	—	—	—	—	*2.38***	*1.21***	.64	1.41
2	*1.96***	*1.13***	*2.39***	*1.45***	**.73**	.71	**.27**	.15	1.10
3	.61	1.56	**.59**	.49	—	**.21**	—	**0**	0.58
4	**1.21**	.31	—	—	—	—	—	—	0.76

Note. For each single problem, paired sample *t* tests were conducted between the loose and the tight chunk conditions. The italicized cells indicate loose chunks; the boldface cells indicate tight chunks. NOC = number of contacts. Dashes indicate no coins with this number of contacts.
* $p < .05$. ** $p < .01$.

comparing Problem F (solvers = 20/28; $M = 0.71$, $SD = 0.46$) with Problems G and H (solvers = 56/58; $M = 0.97$, $SD = 0.13$) revealed a highly significant difference, $\chi^2(1, 84) = 11.12$, $p < .01$, $\lambda = .16$, demonstrating that in Problem F, fewer participants benefited from the 3-D cue than in Problems G and H. One point of note is that Problem F differs from Problems G and H by virtue of having all coins as one group rather than two or more groups (see the NOC column in Table 3). It is possible that this was the cause of the statistical effects seen. However, one should also note that the number of separate groups of coins is reflected in the TNOC variable. Problems that have two or more separate groups of coins will have a lower TNOC by definition and will therefore comprise a greater number of loose chunks. This is suggested in the tendency of higher solution rates for non-3-D-cue problems that have two or more separate groups of coins (Problems D and E) as opposed to those that do not (Problems A, B, and C). In the 3-D cue problems, an interaction between perceptual grouping and 3-D cues exists whereby 3-D cues are more effective when the chunks are loose rather than tight.

Discussion

In the present study, we scrutinized the influence of perceptual grouping and 3-D cues on the solution of the eight-coin problem. The goal was to clarify the main sources of problem difficulty. The results demonstrated that irrespective of whether or not 2-D criterion moves were available (Problem B), move selection and problem difficulty were primarily determined both by perceptual grouping effects (tight vs. loose chunks) and by self-imposed constraints (2-D constraint).

For 3-D cues, the results clearly showed that the presence of a 3-D cue significantly increased solution rates. We also showed that the presence of the 3-D cue increased the likelihood of attempting moves that stacked coins on top of one another (causing higher solution rates). For perceptual grouping, we showed that coins that were part of loose chunks had a much higher likelihood of being manipulated than coins that were part of tight chunks. Loose chunks also facilitated the solution of the 3-D cue problems, with higher solution rates for 3-D problems that had a low TNOC. These findings are in accordance with the assumptions of the RCT.

The fact that 3-D cue problems were easier to solve than non-3-D-cue problems conflicts with the results of Ormerod and colleagues (2002), who found little influence of their visual 3-D hint. One crucial difference between the 3-D cues that were given in the present study and the 3-D cues that were given by Ormerod and colleagues is that our 3-D cue showed coins overlapping with each other, whereas Ormerod and colleagues showed coins that were “flush” with each other. It could therefore be argued that our 3-D cue provides additional information to that of Ormerod and colleagues, in that the solution to the problem involves stacking coins on top of each other such that they only overlap with each other. First, it might be the case that overlapping coins have higher affordances to be manipulated, due to a looser chunking with the

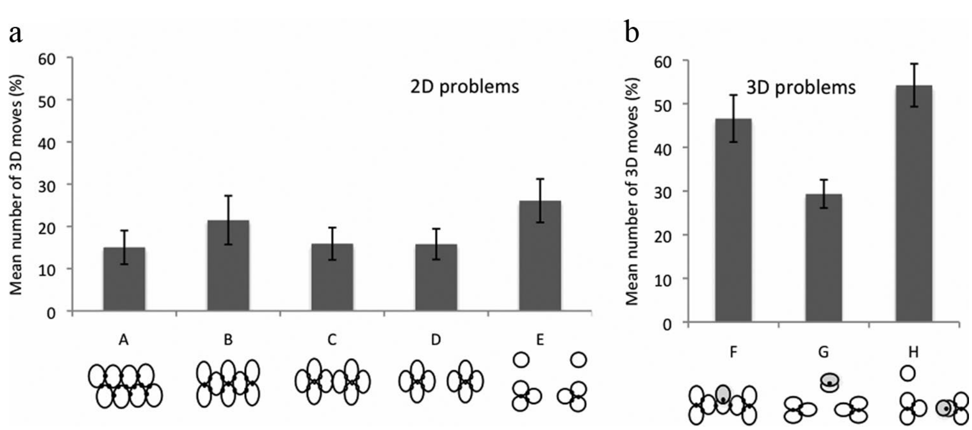


Figure 5. Mean number of moves that end at a 3-D position, with standard error bars.

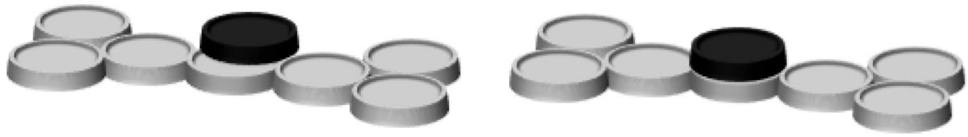


Figure 6. Problem F with two different ways of presenting the 3-D hint: (a) overlapping hint as used in the present study and (b) "flush" coins as used by Ormerod et al. (2002). The 3-D hint coins are colored black for illustration.

other coins. Figure 4 demonstrates that there is a strong preference to manipulate these particular coins. Second, it might also be that overlapping coins trigger a clearer goal representation where the participant realizes that coins need to be overlapping with other coins, as required in the solution; that is, the coin at the top overlaps with three other coins (see Figure 6). This would be an additional source of problem difficulty that was not explicitly addressed in our study and requires further empirical work.

One additional difference across the studies was the use of "coins" (present study) versus hexagonal shapes (Ormerod et al., 2002, study). We would not expect this to be the root cause of differences in performance across 3-D cues. A further examination of the precise aspects of 3-D cues that cause facilitation versus those that do not is therefore warranted.

Our data also showed the role that perceptual grouping plays in insight problem solving. Even for the 3-D cue problems, which were solved quite often, perceptual grouping was still an additional source of problem difficulty, over and above the influence of 3-D cues. Consequently, tight perceptual groupings are an additional source of problem difficulty that hinders the stacking of coins, which are required by the proper solution of the problem. We can therefore suggest that the primary source of difficulty in the eight-coin problem is the self-imposed 2-D constraint, with the decomposition of tight chunks being a secondary source of difficulty that further impeded the solution.

The detailed move analysis showed that, as predicted by the CSPT, participants preferred target positions where a coin touches exactly three others (Problem B, see Figure 4). However, the presence of criterion moves did not cause a reduction in solution rates when compared with problems that did not have any criterion moves available (comparison of Problems A and B in Table 1). Thus, we did not replicate the findings of Ormerod and colleagues' (2002) pilot study that was based on 12 participants in each condition. Given that we used 28 participants per condition in the present study, it would seem that our design provided enough power for a replication. Importantly, our participants' strong preference for initial moves that met the "three coins touching" criterion for Problem B (see Figure 4) did not affect their ability to solve the problem any differently than if they were to solve problems without any criterion moves. Thus, criterion moves were not a factor in the present experiment.

Previous research has suggested that the CSPT may describe the chain of events that occur before a representational change occurs (Jones, 2003; MacGregor et al., 2001; Öllinger et al., 2006). This questions whether the CSPT and the RCT are competing theories or, rather, explaining different phases of the solution process of the eight-coin problem. The CSPT proposes that the search through the problem space is biased toward moves that progress the problem solver the furthest toward his or her goal (i.e., a hill-climbing approach), which leads to failure for insight problems. The RCT proposes that the initial problem space is underrepresented (due to constraints or chunked knowledge) such that the solution to the problem is not a part of the initial problem space. The present data suggest that the RCT alone can explain the difficulty of the eight-coin problem. The effect of the hill-climbing approach subscribed to by the CSPT causes minimal disruption to the insight process in this problem. An additional look at the initial move analysis for Problem B in Figure 4 shows that the end position of moves results in a coin touching three others; however, although participants selected end positions that met the criterion for Problem B, they also chose to manipulate coins that already met three other coins (see Table 3). Thus, participants tried to close perceptual gaps rather than apply a maximization and progress monitoring heuristic. It should be noted, however, that in other problems such as the nine-dot problem, CSPT explains important sources of problems difficulty (Kershaw & Ohlsson, 2004; MacGregor et al., 2001).

In conclusion, we have shown how the 8-coin insight problem can be modified in order to test competing predictions from the CSPT and the RCT. Our data clearly show how constraints are placed on the problem (working in a 2-D space) and how perceptual groupings (tight vs. loose chunks) affect the insight problem-solving process. Within the eight-coin domain, it was clear that the largest obstacle in problem solution was that of constraining oneself to working in only two dimensions, closely followed by that of perceptual groupings. Working to a criterion of satisfactory progress, as per the CSPT, had minimal effects on the solution process. The RCT provides a solid explanation for performance in 8-coin problems across a range of different initial states of the problem.

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