

## The iso-effect: Is there specific learning of Tower of London iso-problems?

A. H. Faber<sup>1</sup>, N. Kühnpast<sup>2</sup>, F. Sürer<sup>3</sup>, A. M. Hinz<sup>4</sup>, and  
A. Danek<sup>3</sup>

<sup>1</sup>Graduate School of Systemic Neurosciences, <sup>2</sup>Master program in Neuro-Cognitive Psychology, <sup>3</sup>Neurologische Klinik und Poliklinik, and <sup>4</sup>Department of Mathematics, Ludwig-Maximilians-Universität, Munich, Germany

The “Tower of London” puzzle was adapted to tablet PCs to be used as a clinical bedside test. “Iso-problems”, a specific class of problems, require identical moves but ball colours are permuted. Thus difficulty is the same even if the appearance is different. We wanted to determine the impact of these as yet little-studied tasks and hypothesised that there may be a learning effect specific to them (the “iso-effect”). We interspersed a set of six iso-problems within one selection of 22 tasks and analysed problem solving by 81 healthy adults (mean age 41.6 years). Participants showed learning across iso-problems (less time, fewer moves, increasingly efficient solutions). This effect was distinct from general learning, as was obvious from comparison with a series of non-isomorphic tasks. However, participants seem not to be aware of solving such problems. This “iso-effect” may be related to implicit memory, a domain that so far has not been assessed using the Tower of London.

**Keywords:** Cognitive skill learning; Implicit learning; Problem solving; Tower puzzle; Transformation task.

Transformation problems (or “look-ahead puzzles”, Shallice, 1982, p. 203) are a common approach to assess executive functions and problem solving. A classic example is the Tower of Hanoi. As an alternative,

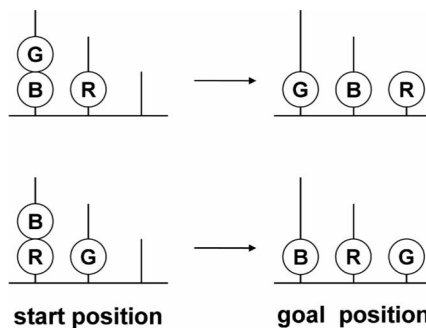
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Correspondence should be addressed to Ms Amory Faber MSc, Dipl-Psych, Graduate School of Systemic Neurosciences Biozentrum LMU, Großhadernerstr. 2, 82152 Martinsried, Germany. E-mail: faber@biologie.uni-muenchen.de

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Shallice designed the Tower of London (ToL) to make “heavy use of a general-purpose planning system” (Shallice, 1982, p. 203). To master the ToL puzzle, the problem solver must rearrange three balls of different colours that are placed on three pegs in an initial configuration to match a certain goal configuration. Despite its popularity for assessing problem solving and the vast literature devoted to it, one interesting ToL aspect has as yet hardly been investigated. Our study focuses on special sets of tasks, so-called iso-problems, first recognised by Berg and Byrd (2002). At issue is whether participants profit from repeated presentation of iso-problems and show improved performance.

ToL tasks that differ from each other only by a permutation of ball colours are mathematically equivalent. The set of all ToL tasks consists of 210 classes of six mutually equivalent tasks each. These so-called “equivalence classes” are disjoint, i.e., they share no common elements. The six elements of an equivalence class have been named “iso-problems” by Berg and Byrd (2002). This was meant to parallel the term “isomorphs”, namely puzzles with identical state graphs but different appearance (Kotovsky, Hayes & Simon, 1985). Hinz and colleagues (Hinz, Kostov, Kneissl, Sürer, & Danek, in press) clarify the issue of isomorphy in the context of tower puzzles from a mathematical point of view (symmetry properties of state graphs) and propose a mathematical model for them. This model is a profound aid for structuring and comparing the many existing tower variants. Iso-problems require identical moves as they have the same start and goal position, but the colours of the balls are permuted (see Figure 1). Iso-problems are from the same equivalence class and therefore have the same level of difficulty, but look different to the observer. Past observations indicate that participants are not aware of their similarity (Johns, 2005); however, very little research has yet been devoted to them.



**Figure 1.** Two of the iso-problems used in this study (problems 12 and 17). Red, blue, and green colours of the balls are indicated by “R”, “B”, and “G”.

The first study that implemented ToL iso-problems was conducted by Ouellet and colleagues (Ouellet, Beauchamp, Owen, & Doyon, 2004). Their participants learned a sequence of repeating problems better than random sequences of problems. However, no particular conclusions regarding iso-problems were drawn because a problem-wise analysis was not performed and there was no direct reference to the unique features of iso-problems. Johns (2005) studied the impact of ToL iso-problems on performance, embedding them in a set of 48 problems. In the course of solving these 48 problems, participants showed a linear trend of improvement in solution times.

Anzai and Simon (1979) studied the basic learning mechanisms that enable humans to improve their performance when repeatedly exposed to the same problem. They proposed a theory of the specific processes of “learning by doing” and showed that past experience with a task does improve performance. Anzai and Simon had recorded one participant’s thinking-aloud protocol during four consecutive attempts at solving the same Tower of Hanoi problem. They characterised the learning process that ultimately leads to the solution of a problem as a gradual progression from less efficient strategies to the most efficient one due to increased familiarity with the task.

The present study systematically examines the impact of ToL iso-problems on performance and assesses participants’ awareness of these problems. At issue is whether participants will be able to improve performance as a result of prior experience with an iso-problem and whether they will recognise the structural similarity.

## METHOD

### Participants

A total of 81 volunteers participated, and none had prior experience with the ToL puzzle. Data were collected in accordance with the ethical standards of the 1964 Declaration of Helsinki by convenience sampling. All participants gave informed written consent. Gender was evenly distributed (42 females, 39 males). All but four participants were right-handed. Age ranged from 20 to 70 years (mean = 41.6,  $SD = 16.03$ ) and all participants had at least 8 years of formal education.

### Procedure

Participants manipulate a game board with the goal of rearranging a set of three balls placed on three pegs from their initial position into another specified configuration. There are certain restrictions as each of the three

pegs of different heights can hold only one, two, or three balls, respectively. Instead of a wooden game board, a computer version of the game on a tablet PC was used (Hinz et al., in press). This computer version is a realistic, three-dimensional reproduction of the original wooden game used by Shallice (1982). The differently coloured balls are moved one at a time from one peg to the other by tapping on the touchscreen with a stylus. Every move is divided into two steps: First, participants pick a ball, which is subsequently highlighted with a coloured frame. Second, to move the ball to another peg, they tap at the peg chosen. The ball is then moved automatically from one peg to the other. For each problem, the minimum number of moves required is shown on the screen and participants are asked to try solving the task with this number of moves.

Participants completed the ToL experiment in one session of approximately 45 minutes. Instructions were given in written form and also explained by the experimenter, followed by two practice trials. All participants were then presented with the 22 problems in the same fixed order. The exact order is provided in the appendix. After solving the last of the 22 problems, the participant was asked “Have you noticed anything concerning the last two problems?” in order to check for the awareness of iso-problems. If the participant’s answer included notions like “Yes, they followed the same principle” or similar, awareness of the iso-problems was assumed (“awareness before cue”). If the participant had not noticed anything special, the experimenter asked: “Have you perhaps noticed that the last two problems were the same, only with different colours?” In this case, a positive answer was coded as “awareness after cue”.

We followed as closely as possible Shallice’s original instructions: At no point is time mentioned in the instructions: if the participant asks specifically “Am I being timed?” the experimenter replies “Please do the problems at your own speed” (Shallice, 1982, personal communication, “Instructions for the Tower of London”). Likewise, when the participant asked how many moves had already been performed, the experimenter answered “I do not count your moves, that is done by the computer.”

### Selection of problems

In addition to the 12 original problems (Shallice, 1982), 10 new problems were implemented. In order to tap the full potential of the ToL, the test battery ranged from two to eight moves. All six iso-problems from one equivalence class were added. Iso-problems with a high minimum number of

moves (6-move problems) were selected because they offer more information through higher variance. The iso-problems selected possess a special feature in that there are three distinct optimal solutions (i.e., shortest paths) for each of them. The complete set of ToL problems thus consisted of the 12 original Shallice problems plus six 6-move iso-problems plus two 7-move and two 8-move problems. Their exact configurations are stated in the appendix.

The experiment ended with two iso-problems in succession, so that participants could be interviewed about awareness of their special character.

### ToL measures

All parameters were automatically recorded by the testing program.

1. "Solution time": Amount of time needed to solve a problem, measured in seconds. It consists of two phases: initiation (for thinking about the problem) and completion (to execute the moves).
2. "Initiation time": Time from presentation of the problem to the first move (i.e., the time needed for planning). In this study the end of initiation time (= beginning of completion time) was defined as the second step (i.e., the participant indicating the target peg) of the first move (Berg & Byrd, 2002). Even when the ball that will be moved has already been selected in the first step, participants can continue planning until a peg is finally chosen.
3. "Completion time": Time span between first and last move.
4. "Moves": Number of moves needed to solve the problem.
5. "Efficiency": Minimum number of moves divided by actual number of moves, between 0 and 1. A value of 1 indicates that the problem was solved optimally.
6. "Awareness" of iso-problems: Participants' answers to the questions regarding the last two iso-problems were coded as: "no awareness", "awareness after cue", and "awareness before cue".

### Data analysis

Data were normally distributed (Kolmogorov-Smirnov test) and hence were analysed with parametric tests using the statistical package SPSS 13.0. All *p*-values are Greenhouse-Geisser corrected. The present analysis focuses on participants' performance on the six iso-problems. They were compared with the help of an ANOVA for repeated measures. The analysis of the 16 problems other than iso-problems is described in Kühnpast and Faber (2006).

## RESULTS

There was a clear trend of improvement from one iso-problem to the next in all performance measures (Table 1).

There was a significant difference between the iso-problems in all dependent variables; namely, solution time,  $F(5, 400) = 9.94$ ,  $p < .01$ , initiation time,  $F(5, 400) = 4.06$ ,  $p < .01$ , completion time,  $F(5, 400) = 6.12$ ,  $p < .01$ , moves,  $F(5, 400) = 3.08$ ,  $p < .05$ , and efficiency,  $F(5, 400) = 3.51$ ,  $p < .01$  (see Figure 2).

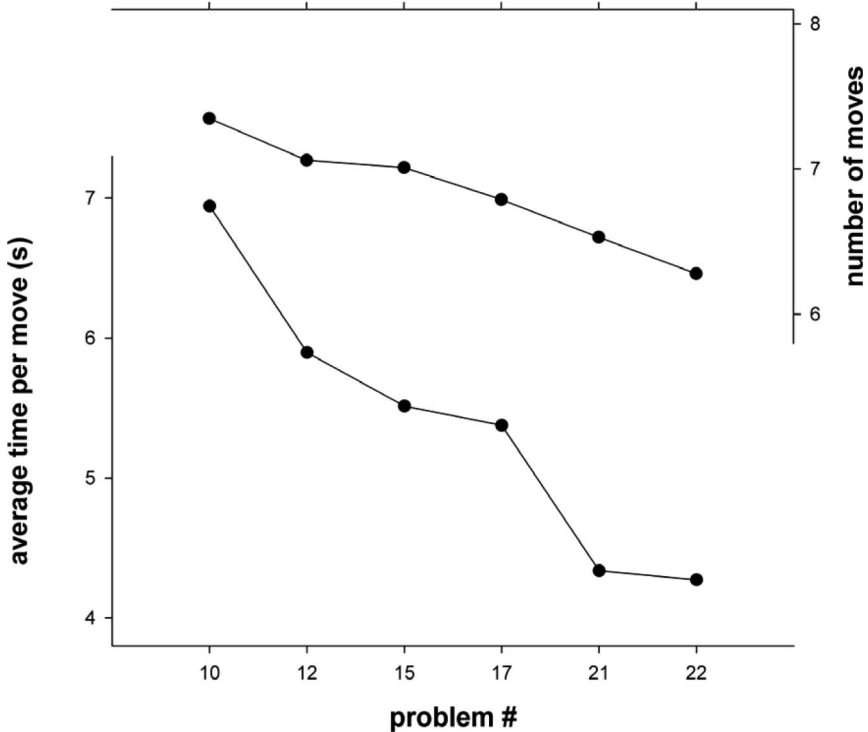
Participants' performance clearly improved from iso-problem to iso-problems, while most of them were not aware of solving iso-problems. Only 27 of the 81 participants were immediately aware of the repetition of iso-problems (i.e., before the cue); 16 more became aware of that fact after they were given the cue and 38 showed no awareness that they had solved iso-problems (neither immediately nor after the cue). Thus, one third of participants were immediately aware of the iso-problem property (33.3%), whereas almost half (46.9%) were not aware of it at all. Please distinguish between awareness before cue (answers to "Have you noticed anything concerning the last two problems?") and awareness after the cue (answers to "Have you perhaps noticed that the last two problems were the same, only with different colours?"). In hindsight, the second question now seems to be put in a rather suggestive manner and we have the impression that some participants simply answered yes in order not to appear unintelligent. Thus, the percentage of participants "aware after cue" (19.8%) does not necessarily reflect the true number of participants with awareness. Because of this we will usually refer to the two thirds of participants who reported no special property of the final two problems when they were asked first.

The effect of improved performance across iso-problems remained unchanged even if the 27 participants who did show awareness (uncued awareness) were excluded from the analysis. The analysis based on only

TABLE 1  
Mean values for the six iso-problems

Problem #	10	12	15	17	21	22
Moves	7.35 (2.6)	7.06 (2.7)	7.01 (2.2)	6.79 (2.2)	6.53 (1.5)	6.28 (1.1)
Solution time	94.22 (58.9)	82.44 (72.8)	81.85 (52.5)	72.96 (45.7)	58.77 (27.5)	55.51 (30.5)
Initiation time	38.43 (34.4)	34.89 (26.9)	39.6 (37.5)	33.93 (28.1)	29.89 (24.0)	27.8 (23.3)
Completion time	55.79 (55.2)	47.56 (67.9)	42.25 (37.2)	39.04 (35.1)	28.88 (14.8)	27.7 (17.4)
Efficiency	0.88 (0.19)	0.92 (0.18)	0.91 (0.18)	0.94 (0.16)	0.95 (0.13)	0.97 (0.1)

SD in brackets. Please note that for efficiency, increasing values represent more efficient solutions (efficiency = 1 for optimal solutions). Time is measured in seconds.

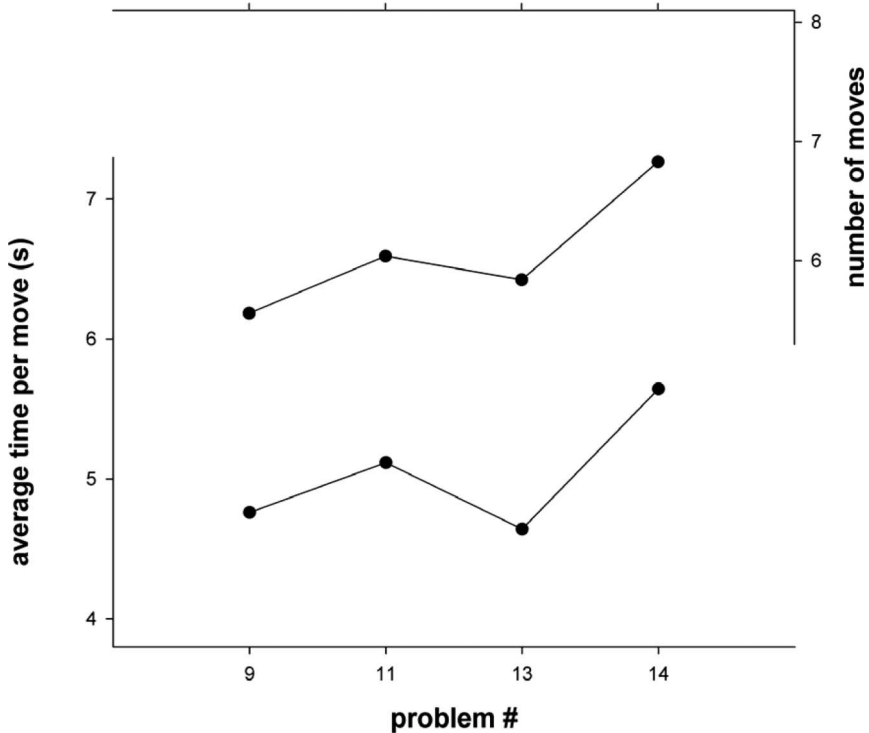


**Figure 2.** Learning curve for the six iso-problems for completion time (lower curve) and moves (upper curve). “Average time per move” (in seconds) results from dividing the completion time by the number of moves for each problem separately.

those participants unaware of the repetition again showed that iso-problems differed significantly from each other for solution time,  $F(5, 265) = 6.14$ ,  $p < .01$ , completion time,  $F(5, 265) = 4.13$ ,  $p < .01$ , moves,  $F(5, 265) = 2.93$ ,  $p < .05$ , and efficiency,  $F(5, 265) = 3.02$ ,  $p < .05$ .

To separate this effect from a general learning effect over all problems, we performed an analogous analysis (now again including all participants) of all four 5-move problems. Apart from initiation time, these also differed significantly from each other in all variables (all  $p < .05$ ). However, the difference was not based on improvement, but instead participants performed less well in the course of solving those problems. The upward sloping curve for 5-move problems clearly contrasts with the downward sloping learning curve found for iso-problems (see Figure 3).

A similar picture results from comparing all 4-move problems, with participants not being able to improve their performance, although in this case all four problems had to be solved in immediate succession (data not shown).



**Figure 3.** Differences between the 5-move problems for completion time (lower curve) and moves (upper curve). “Average time per move” (in seconds) results from dividing the completion time by the number of moves for each problem separately.

We conclude that we did not observe a general learning effect, but rather one that is specific to iso-problems.

## DISCUSSION

In the present study we investigated if ToL performance improves across a series of iso-problems. We find a clear trend of constant improvement in participants’ performance from the first iso-problem to the sixth iso-problem, although other problems are intermixed. This improvement is apparent across all dependent variables (solution, initiation and completion time, moves and efficiency). Considering the fact that only one third of the sample is aware of the repetition of these problems, it can be concluded that participants learn without realising it.

This finding (improved performance from iso-problem to iso-problem without being aware of their similarity) will be called the “iso-effect” in the



following. This learning effect seems to be based on non-declarative knowledge because most participants are not able to report it. There is evidence that cognitive skills can be acquired through implicit learning processes (e.g., Cohen & Squire, 1980). Implicit learning is the ability to gradually acquire a motor skill or a cognitive routine through repeated exposure to a specific activity constrained by invariant rules (Daum et al., 1995). Specifically, the iso-effect could be caused by some sort of implicit “perceptual learning” as proposed by Anzai and Simon in the context of tower puzzles. In their study, the participant arrived at the final problem-solving strategy less systematically and “certainly with less awareness” than in earlier trials (Anzai & Simon, 1979, p. 127). The authors claim that perceptual cues are responsible for perceptual learning. The identical spatial configuration of iso-problems could offer such perceptual cues to the problem solver and therefore allow for a faster solution every time another iso-problem is presented. Although referring not to iso-problems but to the successive solving of identical problems, Anzai and Simon suggest that this process takes place without awareness. Our findings indicate the same.

The iso-effect found in our sample seems to be extremely robust. Plotting the means reveals obvious learning curves for all performance measures. With regard to speed, the improvement in completion time is slightly more pronounced than the one in initiation time. We take this as further evidence that participants are not aware of the repeated presentation of iso-problems, and argue that awareness would be reflected in a much stronger decline of initiation times. If participants had realised that they were solving yet another problem of the same structure, they probably would not have spent any more time on planning. This argument is also confirmed by the fact that the results remain unchanged even if the 27 participants who show awareness are excluded from the analysis. A re-analysis based on only those 54 participants who were unaware of the repetition yielded significant improvement across iso-problems for all variables apart from initiation time. This provides further evidence for the point discussed above. Participants who are not aware of the repetition spend as much time on planning subsequent iso-problems as on earlier ones because they do not realise that they are being presented with the same problem. They only profit from the repetition implicitly, and that is reflected in shorter completion times, fewer moves, and higher efficiency. To sum up, the iso-effect is still apparent even if only participants who were unaware are included in the analysis (please note that this holds true although the statistical power is greatly reduced). This strengthens our proposition of an implicit learning process as one possible basis of the iso-effect.

Some ToL problems can be solved with several alternative optimal solution paths. As stated in the appendix, there exist three optimal solution

pathways for each of the iso-problems. Considering the learning effect across these problems, it is of interest whether participants came to use the same solution paths over repetitions. We therefore qualitatively analysed the pathways that had actually been chosen to solve the iso-problems. This analysis was done for each of the 81 participants. Only optimal solution pathways were taken into account. We found that more than half of the participants (53.1%) chose structurally identical solution pathways for at least four out of the six iso-problems. In particular, across the six iso-problems, six structurally identical pathways were chosen by 7.4% of all participants, five by 24.7%, and four by 21%.

The improvement observed might possibly be caused by the simple fact that participants learn through repeated presentation of any problems by gaining more experience with the ToL puzzle in general. Such a general learning effect has been described by Ouellet et al. (2004) and it has been suggested that it must be differentiated from another, separate learning effect related to repeating sequences of problems. To investigate if the iso-effect could be distinguished from general learning, an analogous comparison of all 5-move problems was conducted. There is no improvement across these problems (compare Figures 2 and 3). This indicates that the improvement in solving the iso-problems is not due to a general learning effect because in this case there should have been a learning curve for the 5-move problems, too.

However, an alternative explanation is possible. It is not clear to what extent the 5-move problems are comparable with each other. Perhaps there is no learning effect because they have different levels of difficulty. The fourth 5-move problem (problem #14) in particular seems to be extremely difficult compared to the others. We reckon that the “flat ending” of this problem may be responsible for that difference. All other 5-move problems used have “tower endings” which are thought to be easier to solve (Berg & Byrd, 2000). In fact, considering problem endings as recommended by Kaller et al. (Kaller, Unterrainer, Rahm, & Halsband, 2004), our four 5-move problems are ordered in increasing difficulty (tower-end, partial-end, flat-end) and this could perhaps account for deterioration of performance across them. In addition, the current study does not allow us to disentangle the influence of the iso-problems’ flat-ending property from their character as iso-problems.

Comparing yet another set of problems with the same number of minimum moves (four 4-move problems), no learning effects were found. Only iso-problems seem to allow for an improved performance that is already manifest in solving a second problem of the same structure. For example, the problem solvers in Johns’ study (2005) needed a much larger number of problems (48 in total) to improve their solution times. To our knowledge, such clear learning effects after solving so few problems have not

yet been described. The beauty of the iso-effect is that participants learn without realising it.

Still, one could argue for another explanation of the iso-effect apart from implicit learning: increased use of a particular first-move strategy that is useful for the iso-problems but not for the other problems used in this study. A comparison of the optimal pathways for all 4-move, 5-move, and iso-problems with the help of the Tower of London graph (Hinz et al., in press) shows that for all six iso-problems it is useful to first move the topmost ball from the left peg to any of the other two pegs. This first-move strategy is also successful for the first two problems with five moves (#9 and #11). However, for problems #13 and #14, this is not the case. Now let us consider the possibility that the improvement from iso-problem to iso-problem is simply based on an increased use of this first-move strategy. In this case, it could be expected that participants who have adopted this strategy will use it for all problems, among them the 4- and 5-move problems. Therefore they should show a better performance for problems #9 and #11 (for which this strategy is successful) than for #13 and #14. However, in the present study problem #13 is solved better, not worse, than #11. Summing up, the first-move strategy just described is useful for all iso-problems and it also leads to optimal solutions for half of the 4-move and 5-move problems. No improvement, however, could be observed in the latter. This alternative explanation of the iso-effect thus seems unlikely.

For further research, we recommend using a control set of problems of the same difficulty as the iso-problems, yet outside their equivalence class (that is, not iso-problems). Such problems are hard to find because it is quite unclear how the difficulty of a ToL problem can be measured. Tunstall (1999) points out that the number of minimum moves alone is a rather poor measure of problem difficulty, and Kaller et al. (2004) suggest that structural parameters like the start and ending configuration, the demand for subgoal generation, and the number of non-optimal solution pathways must be taken into account.

Berg and Byrd (2002) suggest that iso-problems could be an extremely useful tool for repeated assessment. Implementing iso-problems in subsequent testing sessions offers the possibility of comparing participants' performance on functionally identical problems. In the present study most participants are not aware of the similarity of these problems, although the repetition takes place within a very short time-span. Hence in different testing sessions with greater time intervals there would certainly be no awareness of solving the same problems over and over.

In this study a variety of problems including iso-problems and a broad set of performance measures was used in a computerised version of the ToL oriented on the original Shallice puzzle. By systematically implementing iso-problems, we discovered a learning effect the basis of which is not yet clearly

understood but appears related to implicit learning. If corroborated, this study might open a new functional domain, namely implicit memory, that can be assessed with the Tower of London.

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## APPENDIX

Tower of London problems used

<i>Problem number</i>	<i>Start and goal</i>	<i>Overall configuration</i>	<i>Minimum number of moves</i>	<i>Number of optimal solution pathways</i>	<i>Comment</i>
1	RG/B/- → -/GB/R	p → p	2	1	Shallice # 1
2	RG/B/- → G/R/B	p → f	2	1	Shallice # 2
3	RG/B/- → BG/R/-	p → p	3	2	Shallice # 3
4	RG/B/- → G/BR/-	p → p	3	1	Shallice # 4
5	RG/B/- → GR/B/-	p → p	4	2	Shallice # 5
6	RG/B/- → B/GR/-	p → p	4	1	Shallice # 6
7	RG/B/- → BR/-/G	p → p	4	1	Shallice # 7
8	RG/B/- → -/BR/G	p → p	4	1	Shallice # 8
9	RG/B/- → BGR/-/-	p → t	5	2	Shallice # 9
10	RB/G/- → R/B/G	p → f	6	3	iso-problem # 1
11	RG/B/- → GBR/-/-	p → t	5	1	Shallice # 10
12	GB/R/- → G/B/R	p → f	6	3	iso-problem # 2
13	RG/B/- → GB/R/-	p → p	5	1	Shallice # 11
14	RG/B/- → B/R/G	p → f	5	2	Shallice # 12
15	BG/R/- → B/G/R	p → f	6	3	iso-problem # 3
16	G/R/B → -/RG/B	f → p	7	2	
17	BR/G/- → B/R/G	p → f	6	3	iso-problem # 4
18	BRG/-/- → -/BG/R	t → p	8	2	
19	G/RB/- → B/G/R	p → f	7	1	
20	R/B/G → B/R/G	f → f	8	8	
21	RG/B/- → R/G/B	p → f	6	3	iso-problem # 5
22	GR/B/- → G/R/B	p → f	6	3	iso-problem # 6

The Tower of London problems used in the current study, numbered in the order in which they were presented. The second column (“Start and goal”) states initial and final problem configurations. A slash marks the division between pegs and the balls (R, G, and B denote the red, green, and blue ball, respectively) are represented from top to bottom: e.g., RG/B/- is the state with the red on top of the green ball on the left peg, the blue ball on the middle peg, the right peg being empty. Overall configurations are denoted with f=flat, t=tower, and p=partial tower.